

December 14, 1995

LBL- 38044

## Strong $WW$ scattering in unitary gauge <sup>1</sup>

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### Abstract

A method to embed models of strong  $WW$  scattering in unitary gauge amplitudes is presented that eliminates the need for the effective  $W$  approximation (EWA) in the computation of cross sections at high energy colliders. The cross sections obtained from the U-gauge amplitudes include the distributions of the final state fermions in  $ff \rightarrow ffWW$ , which cannot be obtained from the EWA. Since the U-gauge method preserves the interference of the signal and the gauge sector background amplitudes, which is neglected in the EWA, it is more accurate, especially if the latter is comparable to or bigger than the signal, as occurs for instance at small angles because of Coulomb singularities. The method is illustrated for on-shell  $W^+W^+ \rightarrow W^+W^+$  scattering and for  $qq \rightarrow qqW^+W^+$ .

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<sup>1</sup>This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098.

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## Introduction

Electroweak symmetry breaking may be due to a weak or strong force. In the first case there are Higgs bosons lighter than 1 TeV. In the second case there is strong scattering of longitudinally polarized  $W$  bosons at energies  $\sqrt{s_{WW}} \gtrsim 1$  TeV. By measuring  $WW$  scattering in the process  $qq \rightarrow qqWW$  at a high energy collider such as the LHC we will determine definitively which choice nature has made.

The strong  $WW$  scattering cross sections at high energy colliders have traditionally been estimated by combining the use of the equivalence theorem (ET) and the effective W approximation (EWA).[1] The ET[2] represents strong  $W_L W_L$  scattering ( $L$  denotes longitudinal polarization) in terms of the corresponding R-gauge, unphysical Goldstone boson  $ww$  scattering amplitude,

$$\mathcal{M}(W_L(p_1), W_L(p_2), \dots) = \mathcal{M}(w(p_1), w(p_2), \dots)_R + \mathcal{O}\left(g^2, \frac{M_W}{E_i}\right). \quad (1)$$

( $g$  is the weak  $SU(2)_L$  coupling constant) so that a model of Goldstone boson scattering becomes a model for strong gauge boson scattering at high enough energy. Convoluting  $\sigma(W_L W_L \rightarrow W_L W_L)$  obtained from the ET with the effective  $W_L W_L$  luminosity from the EWA[3]

$$\left. \frac{d\mathcal{L}}{dz} \right|_{W_L W_L / qq} = \frac{\alpha_W^2}{16\pi^2} \frac{1}{z} \left[ (1+z) \ln \frac{1}{z} - 2 + 2z \right], \quad (2)$$

where  $z = s_{WW}/s_{qq}$ , the parton subprocess cross section from the ET-EWA method is

$$\sigma(qq \rightarrow qq W_L W_L) = \int dz \frac{d\mathcal{L}}{dz} \sigma(W_L W_L \rightarrow W_L W_L). \quad (3)$$

The total cross section  $\sigma(qq \rightarrow qq WW)$  is obtained by incoherently adding the signal and background cross sections, with the latter obtained from the standard model with a light or massless Higgs boson, say  $m_H \lesssim 100$  GeV.

The EWA is a good approximation for strong  $W_L W_L$  scattering<sup>3</sup> within its domain of applicability, defined by energies  $E \gg M_W$  and scattering angles big enough that Coulomb singularities from photon exchange are not too large. However because the EWA is obtained from a small angle approximation the transverse momenta of the final state jets (and the  $WW$  diboson) are neglected

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<sup>3</sup> For several reasons the EWA is typically less useful for scattering of transverse modes.

and the transverse momentum distributions of the individual  $W$  bosons are distorted. Furthermore, because the EWA neglects the interference between symmetry breaking sector (signal) and gauge sector (background) amplitudes, it may fail if the signal is not much bigger than the background, as occurs for instance near Coulomb singularities. These problems are both addressed by a method presented here in which strong scattering models, formulated as usual in an R-gauge by means of the ET, are “transcribed” to the complete set of U-gauge tree amplitudes, for  $WW \rightarrow WW$  or  $ff \rightarrow ffWW$ . Collider cross sections are then obtained directly from the  $ff \rightarrow ffWW$  amplitudes without resorting to the EWA. The momentum distributions of the final state quanta and the interference terms are automatically retained.

In discussing strong scattering models the term *model* is used advisedly. The models in the literature are intended only to represent the approximate magnitude of strong  $WW$  scattering cross sections. They assume leading partial waves ( $J = 0$  and/or  $J = 1$  depending on the channel) that tend to saturate but not violate unitarity. They are not complete quantum field theories and in particular the unitarization methods typically violate crossing symmetry. These deficiencies do not effect the utility of the models for the purpose for which they are intended, and they are not addressed by the U-gauge method presented here, which merely allows more information to be extracted within the spirit and limitations of the models.

The following sections present the basic idea, illustrate it for on-shell  $W^+W^+ \rightarrow W^+W^+$  scattering, apply it to the collider process  $qq \rightarrow qqW^+W^+$ , and discuss some of the implications.

### The Basic Idea

Consider strong elastic scattering,  $W_L^+W_L^+ \rightarrow W_L^+W_L^+$ . To leading order in the  $SU(2)_L$  coupling constant  $g$  we decompose the amplitude into gauge sector and symmetry-breaking sector contributions,

$$\mathcal{M}_{\text{Total}} = \mathcal{M}_{\text{Gauge}} + \mathcal{M}_{\text{SB}}. \quad (4)$$

The gauge sector contribution is the sum of the 4-point Yang-Mills contact interaction diagram and the photon and  $Z$  boson,  $t$ - and  $u$ -channel exchange diagrams. Each diagram makes a contribution that grows like  $E^4$  where  $E$  is the  $W$  boson center of mass energy. Gauge symmetry ensures that the terms

proportional to  $E^4$  cancel leaving an  $O(E^2)$  contribution, given by

$$\mathcal{M}_{\text{Gauge}} = -g^2 \left(4 - \frac{3}{\rho}\right) \frac{E^2}{M_W^2} + \mathcal{M}'_\gamma + \mathcal{M}'_Z \quad (5)$$

where  $\rho = M_W^2 / \cos^2 \theta_W M_Z^2$  with  $\theta_W$  the weak interaction mixing angle.  $\mathcal{M}'_\gamma$  and  $\mathcal{M}'_Z$  are the residual contributions of the photon and  $Z$  exchange diagrams, of zero'th order in  $E$ . (There is no residual interaction from the contact diagram.) For instance, the residual photon exchange amplitude, which contains the forward and backward Coulomb singularities, has the simple form,

$$\mathcal{M}'_\gamma = -8g^2 \sin^2 \theta_W (\beta^2 E^4) \frac{\beta^2 + (2 + \beta^2) \cos^2 \theta}{ut}. \quad (6)$$

where  $\beta$  and  $\theta$  are the  $W$  velocity and scattering angle in the  $WW$  center of mass, and  $u, t = -2E^2 \beta^2 (1 \pm \cos \theta)$ .

The order  $E^2$  term in equation (5) is the “bad high energy behavior” that would render massive nonabelian gauge theories unrenormalizable were it not cancelled by the Higgs mechanism. The  $O(E^2)$  term is also precisely the low energy theorem amplitude[4],

$$\mathcal{M}_{\text{LET}} = - \left(4 - \frac{3}{\rho}\right) \frac{s}{v^2} = \mathcal{M}_{\text{Gauge}} + O(g^2) \quad (7)$$

where  $M_W = gv/2$  and  $s = 4E^2$ . The argument is simple: if the symmetry breaking force is strong, the quanta of the symmetry breaking sector are heavy,  $M_{SB} \gg M_W$ , and decouple in gauge boson scattering at low energy,  $\mathcal{M}_{SB} \ll \mathcal{M}_{\text{Gauge}}$ . Then the quadratic term in  $\mathcal{M}_{\text{Gauge}}$  dominates  $\mathcal{M}_{\text{Total}}$  for  $M_W^2 \ll E^2 \ll M_{SB}^2$ , which establishes the low energy theorem to order  $g^2$  without using the ET. More familiar R-gauge derivations use the ET and current algebra or an effective Lagrangian[4] to obtain the same result from Goldstone boson scattering.<sup>4</sup>

Even in the U-gauge method the starting point for strong scattering models is the R-gauge Goldstone boson amplitude, since it is in the Goldstone boson amplitude that the strong dynamics is simply expressed, without the cancellations that complicate the gauge boson amplitudes. In general we consider a strong scattering model labeled “X” for the unphysical Goldstone bosons,

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<sup>4</sup> The validity of the ET to all orders in  $g$  is most natural in Landau gauge (see Kilgore[2]), also a natural choice since the  $w$  Goldstone bosons are indeed massless in Landau gauge.

$\mathcal{M}_{\text{Goldstone}}^{\text{X}}(w^+w^+ \rightarrow w^+w^+)$ . Combining the equivalence theorem, equation (1), with equations (4) and (7), we find the model dependent contribution of the symmetry breaking sector in U-gauge,

$$\mathcal{M}_{\text{SB}}^{\text{X}}(W_L W_L) = \mathcal{M}_{\text{Goldstone}}^{\text{X}}(ww) - \mathcal{M}_{\text{LET}}. \quad (8)$$

up to corrections  $\mathcal{O}(g^2, \frac{M_W}{E})$ . We have used the ET to obtain the transcription from the Goldstone boson amplitude  $\mathcal{M}_{\text{Goldstone}}^{\text{X}}$  to the equivalent symmetry breaking sector amplitude  $\mathcal{M}_{\text{SB}}^{\text{X}}$  for physical  $W_L W_L$  gauge boson scattering. The  $\mathcal{O}(g^2, \frac{M_W}{E})$  corrections are inherent in any treatment of strong  $WW$  scattering. Finally the complete gauge boson scattering amplitude is

$$\mathcal{M}^{\text{X}}(W_L W_L) = \mathcal{M}_{\text{Gauge}}(W_L W_L) + \mathcal{M}_{\text{SB}}^{\text{X}}(W_L W_L). \quad (9)$$

#### On-shell $W^+W^+ \rightarrow W^+W^+$ scattering

We illustrate the method for on-shell  $W^+W^+$  scattering, considering the heavy Higgs boson model with  $m_H = 1$  TeV and the K-matrix strong scattering model.

In the Higgs boson model we can compare the cross section obtained from our U-gauge method to the incoherently combined (“EWA”) cross section and to the exact tree-level cross section. The starting point is the Goldstone boson amplitude,

$$\mathcal{M}_{\text{Goldstone}}^{\text{Higgs}}(w^+w^+) = -\frac{m_H^2}{v^2} \frac{t}{t - m_H^2} + (t \rightarrow u). \quad (10)$$

Applying equation (8) with  $\rho = 1$  and  $s \simeq -t - u$  we obtain the U-gauge transcription,

$$\mathcal{M}_{\text{SB}}^{\text{Higgs}}(W_L^+ W_L^+) = -\frac{t}{v^2} \frac{t}{t - m_H^2} + (t \rightarrow u), \quad (11)$$

which differs from the exact U-gauge Higgs exchange amplitude by terms of order  $\mathcal{O}(M_W^2/s)$ .

Figure 1 compares the differential angular cross sections at  $\sqrt{s} = 1$  TeV. The three lower curves represent the results obtained from the incoherent sum ( $|\mathcal{M}_{\text{Goldstone}}^{\text{Higgs}}(ww)|^2 + |\mathcal{M}_{\text{Gauge}}|^2$ ) (dashed line), the coherent sum  $|\mathcal{M}_{\text{SB}}^{\text{Higgs}}(W_L W_L) + \mathcal{M}_{\text{Gauge}}|^2$  (solid line), and the exact tree level cross section (dot-dashed line). At  $\theta = \pi/2$ , the incoherent and coherent approximations differ from the exact tree result by 13 and 7% respectively. In the forward direction,

$\cos\theta = 0.9$ , where the Coulomb singularity is important, the incoherent approximation differs from the exact value by 46% while the coherent approximation agrees to better than 3%.

The K-matrix model is an arbitrary unitarization of the low energy theorem for the  $J = 0$ ,  $I = 2$  partial wave,

$$\mathcal{M}_{\text{Goldstone}}^{\text{K}}(w^+w^+) = -32\pi \frac{x - ix^2}{1 + x^2} \quad (12)$$

where  $x = s/32\pi v^2$ . Like most strong  $W^+W^+$  scattering model amplitudes but unlike the Higgs boson amplitude,  $\mathcal{M}_{\text{Goldstone}}^{\text{K}}(w^+w^+)$  cannot be expressed as a sum of  $t$  and  $u$  channel terms. Applying equation (8), the contribution to the  $W_L W_L$  U-gauge amplitude is

$$\mathcal{M}_{\text{SB}}^{\text{K}}(W_L^+W_L^+) = 32\pi x^2 \frac{x + i}{1 + x^2}. \quad (13)$$

The angular cross sections from  $(|\mathcal{M}_{\text{Goldstone}}^{\text{K}}(ww)|^2 + |\mathcal{M}^{\text{Gauge}}|^2)$  (dashed line) and  $|\mathcal{M}_{\text{SB}}^{\text{K}}(W_L W_L) + \mathcal{M}^{\text{Gauge}}|^2$  (solid line) are displayed in the upper set of curves in figure 1. The two agree to within 5% at  $\theta = \pi/2$  but disagree by 34% at  $\cos\theta = 0.9$  where the incoherent approximation omits the large interference contribution.

For the discussion of  $qq \rightarrow qqWW$  it will be convenient to express equation (13) in terms of an effective  $s$ -channel scalar propagator, which for  $W^+W^+$  would have charge and weak isospin  $Q = I = 2$ . We assign a conventional coupling  $gM_W H_{\text{EFF}}^{--} W_\mu^+ W^{\mu+}$  to this fictitious object. Working only to leading order in  $M_W^2/s$  we define the effective propagator,

$$P_{\text{EFF}}(s) = \frac{-i}{32\pi v^2} \frac{x + i}{1 + x^2}, \quad (14)$$

so that its  $s$ -channel exchange reproduces equation (13).

In general for any model X in which  $\mathcal{M}_{\text{Goldstone}}^{\text{X}}(ww)$  is a function of  $s$  alone, we can define

$$P_{\text{EFF}}(s) = -i \frac{v^2}{s^2} (\mathcal{M}_{\text{Goldstone}}^{\text{X}} - \mathcal{M}_{\text{LET}}) \quad (15)$$

so that the  $s$ -channel exchange reproduces equation (8).

Notice that  $\mathcal{M}_{\text{LET}}(w^+w^+) = -s/v^2$  contributes  $-i/s$  to  $P_{\text{EFF}}$ , corresponding to a massless, scalar ghost. The unphysical singularity is of no concern since

our discussion is manifestly intended only for large values of  $s$ . In fact the apparent  $Q = 2$   $s$ -channel ghost is just an artifact of our choice of an effective  $s$ -channel interaction — it can be viewed as arising from the  $t$  and  $u$ -channel exchanges of a massless (or light, i.e.,  $m \lesssim O(M_W)$ )  $Q = 0$  Higgs scalar propagating with a physical (i.e., non-ghost) sign.<sup>5</sup> Massless scalar exchange and subtraction of  $\mathcal{M}_{\text{LET}}$  are just alternate ways of representing the underlying physics that cancels the “bad” high energy behavior of the massive Yang-Mills interactions. An alternative description of our procedure for transcribing strong  $W^+W^+$  scattering models to U-gauge is to represent the U-gauge symmetry breaking sector by a massless standard model Higgs boson *plus* a  $W_{\mu,L}^+ W_L^{+\mu} W_{\nu,L}^- W_L^{-\nu}$  contact interaction term given by  $\mathcal{M}_{\text{Goldstone}}^X(w^+w^+)$ .

$qq \rightarrow qqW^+W^+$  scattering

The real utility of the U-gauge method is in the application to  $qq \rightarrow qqWW$  scattering, where we avoid the EWA and recover information about the final state that is lost in the EWA. We begin by again decomposing the amplitude into gauge sector and symmetry breaking sector components as in equation (4). Now instead of 5 Feynman diagrams contributing to  $\mathcal{M}^{\text{Gauge}}$  there are  $\sim O(100)$ . For the  $W^+W^+$  channel these include the five diagrams with  $WW$  scattering topology in addition to diagrams in which one or both final state  $W$ ’s are radiated directly from a quark line. These gauge sector diagrams, including the five with  $WW$  scattering topology, are all calculated exactly so that the cancellations among them required by gauge invariance are exactly fulfilled.

In the diagrams with  $WW$  scattering topology the “initial state”  $W$ ’s are virtual, with space-like masses of order  $-q^2 \simeq O(M_W^2)$ . For pure s-wave strong scattering models, such as the K-matrix model, we parameterize the contribution of the symmetry breaking sector by the effective  $s$ -channel propagator  $P_{\text{EFF}}(s)$ , equation (15). We are then extrapolating the symmetry breaking sector contribution to its on-shell value. The error introduced by the extrapolation is  $\sim \frac{s}{v^2} \frac{-q^2}{s} \sim O\left(\frac{M_W^2}{v^2}\right) \sim O(g^2)$ . Essentially the same extrapolation underlies the EWA[5] and a similar one underlies the ET. (In the ET we extrapolate from gauge dependent Goldstone boson masses to  $M_W$ .)

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<sup>5</sup>For  $W^+W^- \rightarrow ZZ$  the contribution of  $\mathcal{M}_{\text{LET}}$  to  $P_{\text{EFF}}(s)$  would correspond to a  $Q = 0$  massless scalar propagating with a physical sign, i.e., a massless Higgs boson.



In the results presented below this prescription is applied to all  $W$  polarization modes. The effect of the  $\mathcal{M}_{\text{Goldstone}}^X$  contact interaction on the  $W_T W_T$  and  $W_L W_T$  scattering amplitudes is of the order of the  $\mathcal{O}(g^2)$  corrections intrinsic to any strong scattering ansatz. That this and other  $\mathcal{O}(g^2)$  approximations introduced by our U-gauge “transcription” are under control is verified by the comparisons given below of cross sections obtained by the EWA and the transcription method.

Figure 2 compares the EWA and transcribed cross sections at the LHC for  $qq \rightarrow qqW^+W^+$  (for all  $W$  polarizations and neglecting quark masses) using the K-matrix model and the 1 TeV Higgs boson model. To simulate an experimentally relevant cross section a rapidity cut  $|\eta_W| < 1.5$  has been imposed. Eight curves are shown. In each case the dashed line is the EWA and the solid line is the U-gauge transcription. The two upper pairs are the total cross sections, while the lower pairs are the “signals” defined by subtracting the standard model cross section with  $m_H = 0$ . The larger signal is for the K-matrix model.

Figure 2 shows that the cross sections from the two methods agree well, to the extent that the lines are not easily distinguished in some cases. This is as expected since the rapidity cut excludes the Coulomb singularity which would have caused them to differ. The total cross sections agree to within a few percent over the range shown, while the signals differ by about 10% at  $M_{WW} = 600$  GeV and then converge to within a few percent as  $M_{WW}$  increases.

### Discussion

The EWA is useful and computationally efficient, but it provides no information on the rapidities and transverse momenta of the final state quark jets nor on the net transverse momentum of the  $WW$  diboson. Since the EWA sets  $p_T(WW) = 0$ , it distorts the transverse momentum distributions of the individual  $W$  bosons and their decay products. The error is small for  $p_T(W) \gg M_W$  but not at moderate  $p_T(W)$ .<sup>6</sup> The correct  $p_T(WW)$  spectrum of each model is automatically provided by the U-gauge transcription method.

The EWA neglects the interference between the gauge sector and symmetry breaking sector amplitudes, so that it can only be reliably applied when one is

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<sup>6</sup>In earlier work this problem was addressed by smearing the EWA cross section with an empirical  $p_T(WW)$  distribution derived from heavy Higgs boson production — see [6].

much bigger than the other. Thus the EWA computation is valid if the signal is much larger than the standard model  $qq \rightarrow qqWW$  background, but not necessarily if signal and background are comparable. Since  $W^+W^+$  is detected in the like-charge lepton final state  $l^+\nu l^+\nu$  and since solid angle coverage is incomplete at any high energy collider, the  $W_L^+W_L^+$  signal interferes not only with the  $W_L^+W_L^+$  background but also with  $W_L^+W_T^+$  and  $W_T^+W_T^+$ . All these interference effects are automatically included when the strong scattering models are embedded directly in the complete set of diagrams for  $ff \rightarrow ffWW$ .

The most serious shortcoming of the EWA is the inability to provide the final state jet distributions needed to evaluate the efficiencies of jet tag and veto strategies. A veto on events containing moderate-to-high  $p_T$  jets at central rapidity effectively suppresses  $qq \rightarrow qqW^+W^+$  standard model backgrounds at little cost to the signal.[7] A tag on higher rapidity, lower  $p_T$  jets may be necessary to suppress the unexpectedly large background to  $W^+W^+ \rightarrow l^+\nu l^+\nu$  from  $W^+Z \rightarrow l^+\nu l^+l^-$  in which the negative lepton escapes detection.[8]

Jet tag and veto efficiencies for strong  $WW$  scattering have been estimated using the complete set of tree diagrams for  $qq \rightarrow qqWW$  in the heavy Higgs boson ( $m_H = 1$  TeV) standard model.[6, 7] However the diboson energy spectrum in strong scattering models is quite different, especially for colliders of sufficient energy to avoid phase space suppression at  $M_{WW} > 1$  TeV. The jet rapidity and transverse momentum spectra and the tag and veto efficiencies then also differ appreciably between strong scattering models and the heavy Higgs boson model. By transcribing the strong scattering models directly into the U-gauge amplitude, we compute the jet distributions directly from the complete set of  $qq \rightarrow qqWW$  tree diagrams, just as is done for Higgs boson models. The  $p_T$  and  $\eta$  distributions of the jets then correctly reflect the differing  $WW$  energy distributions of the various strong scattering models. In future work we will compare the distributions of the heavy Higgs boson and strong scattering models.

Acknowledgements: I wish to thank M.K. Gaillard and K. Hikasa for helpful discussions. This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contracts DE-AC03-76SF00098 and DE-AC02-76CHO3000.

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## Figure Captions

**Figure 1.** Differential angular cross sections for on-shell  $W_L^+ W_L^+$  scattering at  $\sqrt{s} = 1$  TeV. The lower three curves are for the Higgs boson model with  $m_H = 1$  TeV. The upper two curves are for the K-matrix model. In each case the dashed curve is obtained from the incoherent combination of  $\mathcal{M}_{\text{Goldstone}}$  and  $\mathcal{M}_{\text{Gauge}}$  and the solid line is from the coherent gauge boson transcription. The dot-dashed curve is the exact tree-level cross section for the Higgs boson model.

**Figure 2.** Cross sections for  $W^+ W^+$  production via  $qq \rightarrow qq W^+ W^+$  at the LHC with  $|\eta_W| < 1.5$  for the K-matrix and 1 TeV Higgs boson models, computed by the EWA (dashed lines) and the U-gauge method (solid lines). The two upper pairs of curves are total cross sections (all  $W$  polarizations) while the two lower pairs are signal cross sections (predominantly longitudinal polarization) defined by subtraction of the standard model cross section with  $m_H = 0$ . For both signal and background the larger pair of curves is for the K-matrix.



